CHAPTER 10

THE FLAW OF AVERAGES IN LAW AND ACCOUNTING

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CONTENTS

10.1 INTRODUCTION

When adjudicating in the face of uncertainty, courts often replace a range of possible outcomes with a single "average" case. This leads to systematic distortions, which we refer to as the flaw of averages (FOA).[1] Although mathematicians have understood these concepts for at least a century, adapting law to science requires a better understanding of basic principles by courts and lawyers.

This chapter describes the FOA and presents various cases in which courts have ignored it or only partially integrated it. We illustrate how conventional damages studies are subject to the flaw and conclude with suggestions for a more standardized method of illuminating risk and uncertainty that might reduce misunderstandings leading to litigation.

10.2 FLAW OF AVERAGES: DEFINITION

Under its broadest definition, the FOA describes the systematic errors that occur when analysts base calculations on the average (expected value) of uncertain inputs, rather than the entire distribution of possible inputs. We differentiate between two variants of this misunderstanding.

The weak form of the FOA misjudges risk by ignoring the distribution around the correct average value. For example, consider transporting 10 eggs either one by one in individual baskets, or all at once in a single basket. Suppose that a 10 percent chance exists of dropping any particular basket. Then either basket strategy will leave you with an average of nine eggs. However, someone equating these two strategies based on the average outcome of nine remaining eggs would be guilty of the weak form of the FOA. By analyzing the distribution of eggs, it becomes apparent that if you "put all your eggs in one basket," a 10 percent chance exists of losing the entire lot. With the multibasket approach, however, a 99.98 percent chance exists of having at least five eggs at the end of the day.

The strong form of the FOA uses the average case of an input assumption in a calculation and does not derive the correct average for the output. An example involves a drunk wandering down the middle of a busy highway with an average position on the center line. Someone who predicts the future state of the drunk based on his average position errs with the strong form of the FOA, and will claim that the drunk will remain alive. However, his average state is obviously dead.

(a) The Weak Form. The central idea behind the Nobel Prize winning economics of Harry Markowitz in the 1950s recognizes the weak form of the FOA. In optimizing investment decisions, Markowitz argued that average return was insufficient. Instead, he used a combination of both the average return and uncertainty of return to illuminate a risk/return tradeoff.[2] As a consequence, if two investments have the same average return, the one with the lesser risk has more value.

As an intuitive example of this risk adjusted value, imagine the choice between receiving a $500 million out of court settlement in cash or a 50/50 chance of a jury award of $1 billion or zero. Although both choices have the same average value, most organizations would prefer the first. Although economics accepts this risk/return tradeoff, Johnson et al. illustrate a violation of this principle in accounting rules.[3] The example in their 1993 article in *Accounting Horizons* involves receivables from a counterparty that has a 10 percent chance of defaulting. In spite of the risk, generally accepted accounting principles (GAAP) require the holder to book the face value of the receivables, with no offsetting valuation allowance, stating that "it is not probable that an asset has been impaired." For further incompatibilities of GAAP with the FOA, *see* Sam Savage and Marc Van Allen's article.[4]

(b) The Strong Form. The strong form of the FOA involves nonlinear formulas[5] that depend on uncertain inputs. The basic principle states that evaluating such formulas using the average values of the uncertain inputs does not result in the average value of the formula. Mathematicians refer to this result as Jensen's Inequality after Johan Ludwig William Valdemar Jensen, a 19th century self-taught mathematician who worked for the Danish phone company.[6] One can find his inequality in the seminal work on the valuation of stock options published by Fischer Black and Myron Scholes in 1972.[7]

- Linear Calculations: the average of $F(x) = F(\text{average of } x)$ (no bias)
- Convex[8] Calculations: the average of $F(x) > F(\text{average of } x)$ Example: Options:
- Concave Calculations: the average of $F(x) < F(\text{average of } x)$ Example: Restrictions

Exhibit 10-1. Bias Implied by Jensen's Inequality (Strong Form of the Flaw of Averages)

Jensen's Inequality may be understood in terms of a formula, F, which depends on an uncertain number, x. We use $F(x)$ to denote the output of the formula when the input is x. This mathematical result states that the average value of F is not necessarily the value of F based on the average value of x, as summarized in Exhibit 10-1.

The next section frames the concept in more intuitive terms.

(c) Example: Stock Options. We illustrate the FOA using one of the simplest types of options, a European call option. This contract provides the holder with the right, but not the obligation, to purchase a share of a certain stock at a certain price (the strike price) at a certain time in the future (the expiration date). The payoff for this option is as follows:

$$\text{Payoff at Expiration of Call Option} = \text{The Greater of } \$(S - K) \text{ and } \$0 \qquad (1)$$

where

$\$S$ = the stock price at expiration
$\$K$ = the strike price

If the stock price at expiration exceeds the strike price, the holder will exercise the option to purchase the share at $\$K$, and then sell it on the market at $\$S$, resulting in net gain of $\$(S - K)$. If the strike price exceeds the stock price at expiration, the holder abandons the option as worthless.

Now imagine a hypothetical stock, which at expiration could be worth either $\$(K + 2)$ or $\$(K - 2)$ with an average of $\$K$. Then the payoff will equal either \$2 if the stock increases, or \$0 if the stock decreases. Thus we have:

$$\text{Average Payoff} = (\$2 + \$0)/2 = \$1 \qquad (2)$$

To demonstrate the FOA, consider substituting the average price of the stock ($\$K$) directly into Equation 1. The result would be

$$\text{Payoff of the Average Price} = \text{The Greater of } \$(K - K) \text{ and } \$0 = \$0 \qquad (3)$$

Thus, in this context, the *average payoff* exceeds the payoff evaluated at the *average stock price*. The FOA—an error—would be to use the valuation for the average stock price instead of the average payoff.

Although stock market analysts and academicians accept the 1972 work of Black and Scholes, case law has yet to embrace it consistently, as discussed throughout Chapter 11, "Valuing Losses in New Businesses." One can, however, observe the progress of its acceptance in some professional communities. In 1995 (more than two decades after Black and Scholes' initial paper), the Financial Accounting Standards Board (FASB) issued *Statement 123*,[9] which acknowledges the basic principles of option theory: that the fair value of an option "is determined using an option-pricing model that takes into account the stock price at the grant date, the exercise price, the expected life of the option, the volatility of the underlying stock and the expected dividends on it, and the risk-free interest rate over the expected life of the option." The word *volatility* in the statement accounts for the distribution of all possible stock prices. Using volatility in the valuation can save the analysis from the error of the FOA. Ignoring volatility will lead to the errors of the FOA.

For accounting purposes, however, firms continued to claim that the options it awarded as performance incentives had no cost, and therefore the award did not increase expense or reduce the company's book value. In 2004, FASB issued a revision of *Statement 123*[10] that requires firms to use "the fair value method" when costing share-based compensation. FASB did not specify the type of model to use but suggested choices based on the original Black-Scholes formula, or the more flexible binomial lattice model proposed by Cox et al. in 1979,[11] or the most general form of modeling uncertainty, Monte Carlo simulation (discussed in Section 10.6 of this chapter and Chapter 11).

Not all segments of the business and legal communities have agreed on how to account for the value of stock options, but after a third of a century, it looks like Jensen's Inequality has won at least this battle. Section 10.2.d.(ii) presents an analogous accounting issue in which current practice violates the FOA.

(d) Three Important Cases of the Strong Form. Jensen's Inequality in litigation typically falls into one of three classes:[12] (1) linear calculations, (2) those with real options, and (3) those with real restrictions.

(i) Linear Formulas. When a formula involves only simple sums of uncertain quantities, it is mathematically valid to substitute F(average of x) for the average of $F(x)$. Suppose, for example, that a court has awarded a plaintiff an amount equal to five percent of future dollar sales on some invention in three distinct markets, A, B, and C. Suppose further that both sides agree that the average sales in the three regions are expected to be Avg.(A), Avg.(B), and Avg.(C), respectively. Then it is valid to estimate total average sales as Avg.(A) + Avg.(B) + Avg.(C). Although this computes the correct average, it does not indicate the degree of uncertainty of the sum.[13] In other words, the weak form of the FOA still applies to the linear case.

(ii) Real Options. If a formula, which we will assume measures a benefit, takes into account the decision maker's ability to make a future decision after the resolution of an uncertainty, then the average value of the formula exceeds the value of the formula evaluated at the average input—that is, the average of $F(x) > F$(the average of x). The stock option hypothetical of Section 10.2(c) presented this situation; the future decision of whether to buy or abandon the option can await the actual outcome, where the decision maker knows the actual share price on exercise date.

Consider a hypothetical case in which the court rules that a plaintiff should receive liquidating payment for a share of a mineral mine. The court will base damages on the mine's value. Both sides agree that the operation can recover one million tons of the mineral and that the incremental cost of mining the material equals $10 per ton. Both sides also agree that the mineral has an uncertain future price, ranging between $4 and $20 per ton, with an average of $12. The defense argues that the court should base damages on the average price (i.e., F(average of x)), leading to a valuation of $2 million (= 1,000,000 tons × ($12 − $10)/ton). The average value of the property exceeds this. If the mineral price drops below $10, the mine has the option to shut down, limiting costs when the low price precludes extraction. On the other hand, if the price rises, no such limitation exists on the upside.

A numerical example demonstrates this asymmetry. Imagine that the mineral has only two equally likely possible future prices: $6 and $18, with an average of $12. If the $6 price appears, then the mine has value of $0 because the operators have the option to shut down rather than lose $4 per ton. On the other hand, the $18 price results in a mine value of $8 million. The average of $0 and $8 million equals $4 million, twice the valuation arrived at by using the average price of $12.

In reality, a trajectory of uncertain prices would exist through time, interspersed with a sequence of production decisions. However, the basic principle remains: the option to halt production ensures a greater value than the value based on average price. Viewed from this perspective, the mine becomes a call option on the mineral, with a strike price equal to the marginal production cost. Thus, one might expect the fair value method of FASB *Statement 123* to apply to its valuation, but it does not. When the SEC addressed a parallel situation related to petroleum reserves, a recent *Wall Street Journal* article reported that "[t]he SEC [Securities and Exchange Commission] stipulates that their evaluations must be based on a snapshot of oil prices at the companies' year end, usually Dec. 31."[14] Given the high volatility of oil price and the expected indefinite, but long, life of an oil reserve, the SEC regulation results in undervaluations for such properties. Jeff Strnad of the Stanford Law School has shown that the inconsistency of tax laws pertaining to the real options underlying petroleum exploration can have an adverse effect on energy development.[15]

Note that in real options, the greater the uncertainty in price, the greater the option value, because now the profit can go even further up but it still can't fall below zero. Similarly in the Black-Scholes option formula: the value of a call option goes up with increasing uncertainty (volatility) in the stock price. It is ironic that when valuing lost profits for a new business (*see* Chapter 11), courts will argue that there is too much uncertainty for there to be any damages, which flies in the face of the fact that the more uncertainty, the more valuable the business can be under the right circumstances. We finish this example by considering a modification in which the price is $12 with certainty, but the future marginal cost of mining is uncertain with an average of $10. If the cost exceeds $12, the mine will halt production, but if the cost is low, the profits will be considerable. Again, the option ensures that the average value of the mine exceeds the value of the mine given the average cost of $10. Estimating average damages by substituting average values of the underlying uncertainties leads to wrong results.

(iii) Restrictions Just as the real option to act on future knowledge has a value, a restriction of one's actions imposes a cost. Consider a hypothetical litigation that bases damages on one year's profit derived from a manufacturing plant that produces custom-machined parts on an as-ordered basis. Both sides agree that the plant generates a profit per part of $10, and has the capacity to produce 100,000 parts per year. They also agree that the average demand for parts is 100,000 units but that demand could vary between 50,000 and 150,000. The plaintiff argues that

because average demand is 100,000, the profit formula for damages (or F(average of x)) should equal \$1 million (= \$10/part × 100,000 parts). In this case, the average profit will be less than \$1 million, (i.e., the average of $F(x) < F$(the average of x). Suppose that demand had an equal likelihood of either 60,000 or 140,000 units, with an average of 100,000. The plant's capacity limits production to either 60,000 or 100,000 for an average of 80,000. Thus, average profit would equal \$800,000 (= \$10/part × 80,000 parts), not \$1 million.

(iv) Summary of the Strong Form of the Flaw of Averages The examples of real options and restrictions show that the error inherent in the FOA can result in either over- or undervaluation. Also, it does not require a complex calculation to identify the direction of bias, as summarized earlier in Exhibit 10-1. With real options, the average value is *greater or equal to* the value associated with the average inputs. With restrictions, the average value is *less than or equal* to the value associated with the average inputs.

10.3 FLAWED JUDICIAL DECISIONS

(a) Historical Trends. For years, courts have succumbed to the FOA when valuing financial options. As Section 10.2(c) discussed, financial options have *more* worth than the difference between the market price of the underlying stock and the option strike price. In the legal system, however, "the relevant black-letter rule to measure damages ... [for] failure to deliver publicly traded securities subject to an option agreement... [is] the difference between the public market price of the underlying stock on the date of the breach and the option [strike] price specified in the agreement."[16]

For example, over a decade after the Black-Scholes work, in *Richardson v. Richardson*,[17] the court was attempting to value options "to purchase 3,000 shares of the common stock of the Murphy Oil Corporation." The strike price on the options was \$13.71 per share and "[o]n the day the case was heard Murphy Oil Corporation's common stock was being traded at \$22.50."[18] Based on these facts, the court reasoned that the option holder "could have exercised the options for \$41,130 and received Murphy Oil stock worth \$67,500." As a result, the court *wrongly* "found that the value of the options was the difference between the cost of exercising them and the worth of the stock."[19]

(b) A New Direction. In several recent cases, courts have recognized the FOA when valuing options.[20] The facts in *Custom Chrome* are instructive. Custom Chrome (a supplier of Harley-Davidson motorcycle parts) borrowed \$26 million from the bank. To obtain the loan, Custom Chrome agreed to issue warrants (in essence, options) to the bank to purchase approximately 12 percent of the stock of Custom Chrome. The strike price was \$500 per share, which reflected the estimated market value of the Custom Chrome shares on the day Custom Chrome granted them to the lender.

Custom Chrome petitioned the tax court for a determination as to its right to deduct \$3 million as a financing expense because of the option value of the warrants. However, the tax court found that the warrants had no value at the time of issuance because any future value was highly speculative and the warrants were "at the money" (i.e., at the same price as the stock itself).

After losing the issue before the tax court, Custom Chrome appealed. The Ninth Circuit Court of Appeals agreed with the taxpayer and reversed the ruling of the Tax Court. In reaching this decision, the Ninth Circuit found that "the Tax Court's reasoning and the Commissioner's arguments proffered in favor of a zero valuation are unsupportable."[21]

First, the Ninth Circuit noted: "The Commissioner has cited no authority for the proposition that the lack of a well-defined present market value and the uncertainty in the future value of a financial instrument imply that the instrument has no value for tax purposes."[22] *Second*, and more fundamentally, the Ninth Circuit concluded that an option that is "at the money" does not render the option valueless. The court determined that an option has two values: an intrinsic value and a time value and explained that

> the intrinsic value of an option is the difference between the actual value of a share and the exercise price of the option.... The time value reflects the expectation that, prior to expiration, the price of the stock will increase by an amount that will enable an investor to sell or exercise the option at a profit.[23]

With respect to intrinsic value, the Ninth Circuit observed that "because the strike price of the warrants was the same as the nominal trading price at the time of the buyout, the warrants had no nominal intrinsic value."[24] With respect to time value, the court decided that "the warrants had a substantial time value—that is the bank and everyone else expected the stock price to increase significantly so that eventually it would be profitable to exercise the warrants."[25]

Although the Ninth Circuit takes a step in the right direction, its analysis still falls short of the mark. To have time value, such an option needs only an expectation that the stock price will fluctuate, not increase. Thus, the court used a faulty argument to reach its decision, which would have led to an incorrect result if *the bank and everyone else had expected the stock price to fluctuate around its current price but not increase significantly.*

In *Alliant Energy*, the Seventh Circuit extended the valuation concepts for financial options to real options. In *Alliant Energy*, the plaintiff utility sued the State of Wisconsin for damages resulting from Wisconsin's utility regulations that prohibit a utility from selling a 10 percent block of its stock without regulatory approval. The utility argued that this regulation increased its cost of capital. The Seventh Circuit agreed, ruling that this regulation did increase the cost of capital for the utility because it "deprive[d] the firm of an *option* value—that is, of the power to sell a 10% block." (emphasis added) The Seventh Circuit explained:

> Suppose that there is only a 10% chance that the firm would be able to attract an investor willing to buy a 10% block of stock ... [and] the firm will save $1 million compared with the outlay in hiring the same capital from banks. Then the loss from not having *the option* to make the placement is $100,000, and this loss affords standing.[26] (emphasis added)

The Seventh Circuit reasoned that the option to sell 10 percent of one's stock "has a positive value even if *no one* wants to buy today" (italics in the original) and concluded "[a] firm with the ability to sell such blocks in the future ... is worth more in the market today than a firm hamstrung by laws cutting off its opportunities."[27]

10.4 FLAWED ANALYSIS OF THE VALUE OF LITIGATION

Even in lawsuits in which the court does not adjudicate issues involving the FOA, the parties may nevertheless need to address it. Many people assume that a party would litigate only if it anticipated a positive expected (average) outcome. This assumption ignores the FOA. Bebchuk[28] and Grundfest and Huang[29] have shown that the plaintiff might bring a lawsuit even when the expected outcome is negative. This occurs when the suing party has the option to abandon the litigation, as with the following scenario.

Suppose Party A believes that Party B might sue for average damages known by both sides to be $100,000. Also assume that both sides estimate the average cost of litigation to Party B to be $140,000. Then it would appear that B poses no credible threat to A, and stonewalling would be A's rational strategy. Now suppose that B can divide its litigation costs into two segments of $70,000 each, with the option to abandon the litigation after the first half. Then by spending the first $70,000, B has placed itself in a position in which spending the second $70,000 will result in average incremental return of $100,000. Thus, after the first desperate act, B now poses a credible threat, which should motivate A to settle in the first place.

This line of negotiation is reminiscent of the example described by Roger Fisher and William Ury in *Getting to Yes.*[30] Two drivers are playing chicken by speeding toward each other on a narrow road, with the winner being the one who swerves or breaks last. Suddenly one of the drivers throws his steering wheel out the window in clear view of the other driver, ensuring that his worst outcome is a draw.

10.5 DISCRETE EVENTS

The flaw of averages is common in estimating the outcomes of discrete chance events that either do or do not occur. This occurs, among other places, in accounting for contingent losses as a result of legal action or in valuing losses for a new business (*see* Chapter 11). We will address contingent losses first.

(a) Accounting for Contingencies. In *Statement No. 5, Accounting for Contingencies,*[31] FASB provides guidelines so fuzzy that they do not warrant the distinction of either the weak or strong form of the flaw of averages. It states that "the estimated loss from a contingency be accrued by a charge to income if it is probable that a liability has been incurred and the amount of the loss can be reasonably estimated," without providing a definition of either *probable* or *reasonably* Even if FASB defined *probable*, the idea of a threshold is ill advised. Suppose we define 51 percent to be probable. Then Firm A, which had a pending legal action with a 50 percent chance of a $100,000 judgment, would have no charge, whereas Firm B, with a 51 percent chance, would have a $100,000 charge. Furthermore, Firm C, with 1,000 independent cases of $100, each with a 50 percent chance of judgment, would have no charge. If, instead of a threshold, courts used the correct average (judgments weighted by the probability of occurrence), they would calculate the results as follows for the three firms:

Firm	Calculation	Average Amount
A	50% × $100,000	$50,000
B	51% × $100,000	$51,000
C	50% × 1,000 × $100	$50,000

However, this still commits the weak form of the FOA. Both Firms A and B have essentially even odds of judgments of either $100,000 or zero, whereas Firm C is certain to pay out close to $50,000 in judgments.

(b) Lost Profits from a Risky Venture. Now we will consider valuing lost profits in a new privately held business with a highly uncertain future. A pharmaceutical firm has licensed a compound with which it has been able to cure all known forms of cancer in rats. Credible experts agree that there is one chance in ten that the Food and Drug Administration (FDA) will approve the resulting drug; if it does, the result will be $1 billion of income over the next five years. Otherwise, there will be no profits.

Just before the licensee submits the drug to the FDA, the licensor breaches the contract. Various cases cited in Chapter 11 point to different evaluations of lost profits in similar situations. The damages might be evaluated based on the most likely outcome, that the FDA does not approve the drug. This results in zero damages. Or the court might require a threshold probability of success (which would no doubt exceed 10 percent) before awarding damages, and again the result would be zero damages. The approach more consistent with modern finance theory takes the average profit of 10 percent of $1 billion (= $100 million) and then applies a risk discount factor appropriate for an investment with this level of risk. We will refer to this as the *market valuation*, because it would reflect the share price if the firm were publicly traded. We would argue that zero is too low and that market valuation is too high.

(c) The Certainty Equivalent. Instead, we propose a value that will most often lie between the two, based on the certainty equivalent value of the business. The certainty equivalent value of the business is the value at which the shareholders would have sold out before the contract was breached. Surely the plaintiff in the breach could not ask for more than this. To illustrate the concept of certainty equivalent, suppose you meet a stranger on the way home from work who offers the following gamble based on a coin toss. Heads you get $10 million, tails you get nothing, for an average gain of $5 million. Before flipping the coin, the stranger asks if you would take $50,000 in cold cash to forgo the flip. If your answer is yes, then your certainty equivalent for this gamble is less than or equal to $50,000. If you still want to go ahead with the flip, then your certainty equivalent is more than $50,000. The dollar amount at which you would be indifferent to taking the cash or proceeding with the gamble is defined as the certain equivalent. This depends on the decision maker's risk tolerance. In ascertaining your own certainty equivalent ($*C*) for the above gamble, you can find it useful to imagine losing the toss and then having to explain to your spouse or significant other that you turned down $*C* for the equal chance of $10 million or nothing.

(d) Certainty Equivalent Must Be Less than Market Value for Firm Trying to Go Public. If the new firm has stated its intention to go public, then its shareholders believe that the market valuation (the average risk discounted cash flow stream) would exceed the value of the company while privately held. Therefore, had it been offered a buyout before getting to flip the coin with the FDA, it would have settled on less than the public market valuation.[32]

10.6 MONTE CARLO SIMULATION: A POTENTIAL CURE FOR THE FLAW OF AVERAGES

As we have demonstrated, representing an uncertain quantity with a single average number in a calculation may lead to the FOA. To avoid this problem, one must analyze the calculation relative to the joint probability distribution of the uncertain numbers in question. The most general approach to this problem involves drawing numbers repeatedly from the distribution and running them through the calculation in question to reveal the full range of possible outcomes. The mathematician Stanislaw Ulam developed this approach, referred to as *Monte Carlo simulation*, while working on the Manhattan Project. As its name implies, the approach resembles testing a gambling strategy by repeating it over thousands of rolls of dice. Section 11.5(c)(i) explains the mechanics of a Monte Carlo simulation. Financial analysts and academics use it today to test investment strategies under economic uncertainties.

Orange County, California should have used Monte Carlo in the summer of 1994. Interest rates were low and were expected to remain so or fall even further. The county had created such a successful financial portfolio (assuming this expected future behavior) that they had to turn investors away. Had the portfolio managers explicitly considered the distribution of possible interest rate outcomes instead of a single average scenario, they would have likely foreseen the possibility of the now famous adverse change in rates that forced them into insolvency a few months later. Shortly after the debacle, Philippe Jorian[33] of the University of California, Irvine performed a Monte Carlo simulation based on data available in advance that indicated a five percent chance of a $1 billion loss or worse. Exhibit 10-2 presents a schematic of this process.

10.7 PROBABILITY MANAGEMENT

The FOA implies that managers require distributions of strategic metrics, such as Gross Domestic Product (GDP) growth and oil prices, to effectively manage their firms. These managers require distributions to perform their jobs well. Fortunately, plenty of statisticians and econometricians know how to produce distributions.

Probability Management[34, 35] represents an emerging managerial discipline that will allow those with statistical expertise to "certify" important probability distributions and distribute them to others. William F. Sharpe, who shared the Nobel Prize with Markowitz and Miller in 1990, pioneered this area. In 1997, Sharpe founded Financial Engines,[36] a firm devoted to the Monte Carlo simulation of pension funds. By allowing employees at participating firms to visualize the full range of future trajectories of their pensions over time, including the unfortunate outcomes, this sort of simulation should ultimately reduce the likelihood of litigation. Financial Engines has developed a database of distributions of thousands of mutual funds.

We believe that a standard-setting organization based on a generalization of these ideas could maintain libraries of benchmark distributions for strategic uncertainties such as GDP growth, interest rate, oil price, and so on. These could prove useful for evaluating contended properties, settling litigation, and detecting risky management procedures in advance.

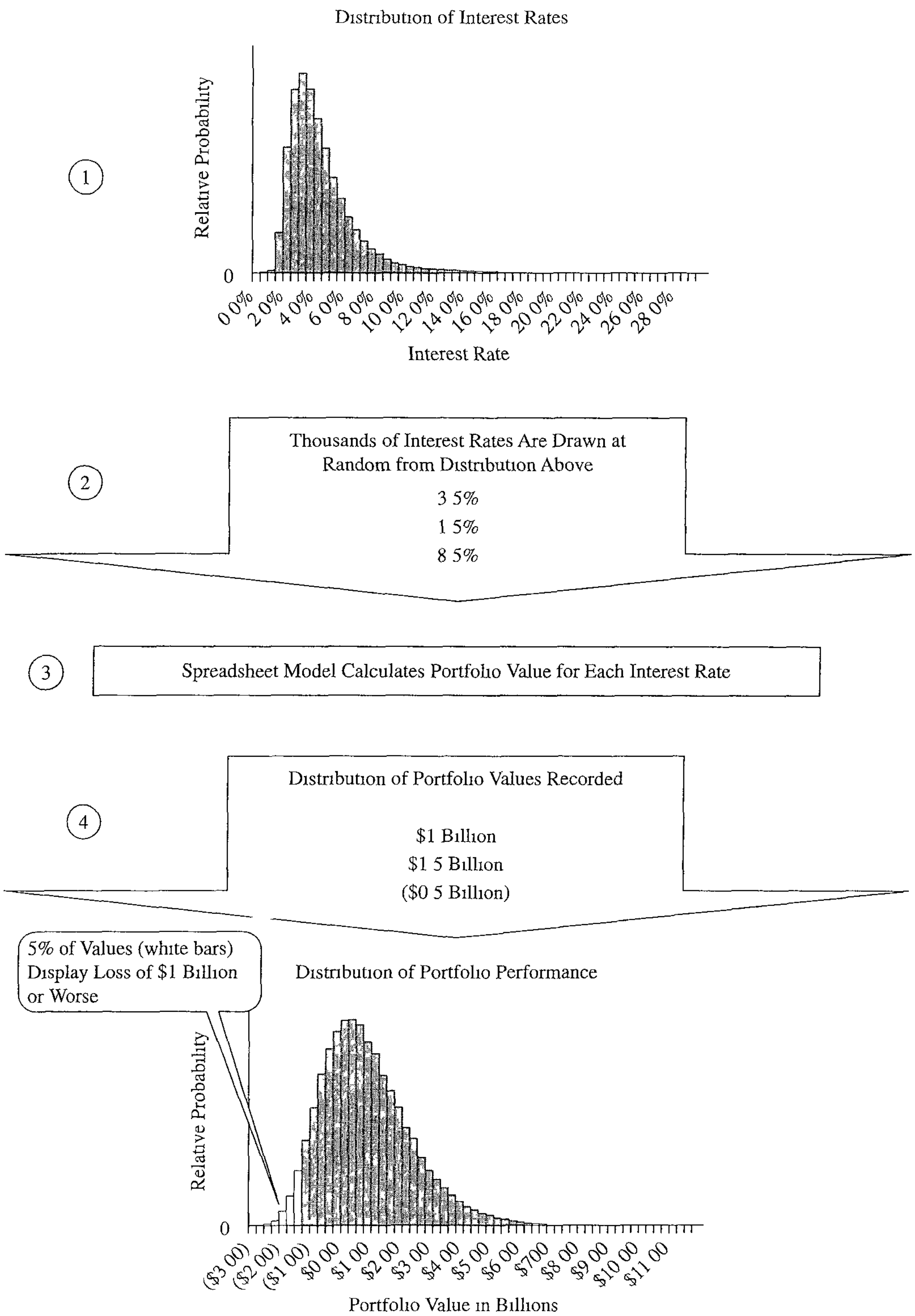

Exhibit 10-2. Monte Carlo Simulation

NOTES

1 S L Savage, "The Flaw of Averages," *Harvard Busmess Revtew* (November 2002)
2 H M Markowitz, *Portfoho Selectton, Efficient Dwerstficatton of Investment* (Hoboken, NJ John Wiley & Sons, 1959)
3 L Todd Johnson, Barry P Robbms, Robert J Swiermga, and Roman L Weil, "Expected Values m Fmancial Reportmg," *Accountmg Horizons* 7 (December 1993) 77–90
4 S L Savage and M Van Allen, "Accountmg for Uncertainty," *Journal of Portfoho Management* (Fall 2002)
5 A formula is said to be nonlinear if its mputs (also known as arguments) enter the formula in a manner other than simple addition or subtraction The nonlinear formula in the example of the drunk above is known as a step function, if he steps to the left he gets killed, and if he steps to the right he also gets killed
6 http //www-groups dcs st-and ac uk/~history/Mathematicians/Jensen html
7 Fischer Black, and Myron Scholes, "The Valuation of Option Contracts and a Test of Market Efficiency," *Journal of Fmance* 27 (1972) 399-418
8 A convex function has a graph that curves up (∪), while a concave graph curves down (∩) The reader need not understand the meanings of convex and concave nor the direction of the inequality to get the main point *F* of average does not equal average of *F* unless *F* is linear
9 http //www fasb org/st/summary/stsum123 shtml
10 http //www fasb org/st/summary/stsum123r shtml
11 John Cox, Stephen Ross, and Mark Rubinstein, "Option Pricing A Simplified Approach," *Journal of Fmancial Economics* 7(1979) 229
12 This characterization is modeled on that of Stefan Scholtes of the University of Cambridge Judge Business school
13 Assume the court asks, "What is the 90-percent confidence interval for sales?" The sum of the averages sheds no light on this To answer this question, one would need to estimate the distribution of possible sales in each region and the extent of the market's statistical independence This would provide an estimate of the overall degree of uncertainty in the award
14 T Carlisle, "How Lowly Bitumen Is Biting Oil Reserve Tallies," *Wall Street Journal*, February 14, 2005
15 Jeff Strnad, "Taxes and Nonrenewable Resources The Impact on Exploration and Development," *SMU Law Review* 55 (2002) 1683–1752
16 *Mathias v Jacobs*, 238 F Supp 2d 556, 575 (S D N Y 2002) (citing *Hermanowski v Acton Corp* 729 f 2d 921, 922 (2d Cir 1984), *Simon v Electrospace Corp* 28 N Y 2d 136 (1971))
17 *Richardson v Richardson*, 280 Ark 498, 502 (1983)
18 *Id*
19 *Id*
20 *See*, e g , *Custom Chrome v Commissioner*, 217 F 3d 1117 (9th Cir 2000) (valuing "financial" options), *Alliant Energy Corp v Bie*, 277 F 3d 916 (7th Cir 2002) (valuing "real" options)
21 *Custom Chrome v Commissioner*, 217 F 3d 1117 (9th Cir 2000)
22 *Id*
23 *Id*
24 *Id*
25 *Id*
26 *Alliant Energy Corp v Bie*, 277 F 3d 916 (7th Cir 2002), at 921
27 *Id*
28 Lucian Arye Bebchuk, "Suites with Negative Expected Value," Discussion Paper No 226 12/1997, in *The New Palgrave Dictionary of Economics and the Law* (Cambridge, MA Harvard Law School, 1998, 551–554

29 J Grundfest, and P Huang, "The Unexpected Value of Litigation: A Real Options Perspective," *Stanford Law School Review*, Volume 58, No. 5 March 2006, pp.1267-1336.

30. Roger Fisher and William Ury, *Getting to Yes: Negotiating Agreement without Giving In* (New York: Penguin Books, 1983).

31. Joel G. Siegel, and Marc. Levine, *GAAP 2002 Handbook of Policies and Procedures*, (Englewood Cliffs, NJ· Prentice Hall, 2002).

32. If the firm's shares were publicly traded and the firm had time to announce the impending gamble it was about to undertake, modern finance suggests that market price of the shares would rise to reflect the expected value, whereas the holders of the shares would sell off enough to reduce their investment in the uncertain gamble to acceptable levels

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